Epochs in Market Sector Index Data - Empirical or Optimistic?

G. Keogh, S. Sharifi, H. Ruskin, M. Crane

School of Computer Applications,
Dublin City University,
Dublin, Ireland

Abstract. We introduce here the concept of "epochs" in market movements (i.e. periods of co-movements of stocks). These periods in EURO-STOXX market sector data are characterised by linear relationships between price and eigenvalue change. The evidence suggests strong time dependence in the linear model coefficients but residuals are strongly dependent on granularity (i.e. sampling rate) with fit breaking down at rates smaller than five days. Possible reasons for this breakdown are presented together with additional arguments on the relative merits of correlation and variance-covariance matrix eigenanalyses in measuring co-movements of stocks.

1 INTRODUCTION

The leverage effect (i.e. the fact that at-the-money volatilities tend to increase for asset price drops) in financial markets has been much studied over recent years. This increase has been shown to apply for different forecasting horizons, dependent on whether studies focus on the volatility in auto-correlations of actual stock prices or on those of index data. The literature is reticent, however, about the effect of price changes on the combined upward or downward movements of shares and it is this aspect which we address. Specifically, we have developed, for different market sector data from Dow Jones EURO STOXX, a novel approach to the way in which the market recognises and responds to risk. This, implicitly, reflects the volatility, where risk reaction clearly changes as prices rise or fall or crashes occur. The method is based on examining change in the riskiest position, as determined by the maximum eigenvalue from an eigenanalysis of the variance-covariance matrix from day-to-day, and relating this to effects on prices for the various sectors. The change in the largest eigenvalue acts, in some sense, as a barometer of market risk.

Results to date indicate that there are periods in the sector data, for which change in the largest eigenvalue varies linearly with price. As each period appears to end, the relationship changes and a smaller change in the eigenvalue is required to bring about the same change in price. In the limit and during a crash, the slope is still positive (and quite large). This is evidence, we believe, that crash patterns are well-defined, due to common perception of risk change, while upward trends are less predictable. Evidence tends to support an implicit relationship between the instantaneous volatility and the daily changes in the eigenvalues, not least because alternative analysis of the correlation matrix, (for the data normalised with respect to the variance), results in a disappearance of these periods. The above effects vary from sector to sector, being seen to be most pronounced in the technology and telecoms sectors and less so in more defensive sectors.

In summary, evidence of periods in eigenvalue-price correlations is presented together with additional arguments on the relative merits of correlation and variance-covariance matrix analyses in measuring co-movements of stocks.

2 THE BASICS

In order to make money (or more euphemistically "maximise their utility") on stock markets, investors buy and sell assets. As the overall risk associated with a portfolio of stocks can be shown [Elton et al.(2002)] to decrease with the number of assets, more is better when it comes to assets. However, by having more assets, the investor will potentially take on more risk in order to generate higher expected returns. A balance needs to be struck, therefore, between the risk a new asset will add to the portfolio and the expected return. This problem of balance requires a knowledge of the volatility of and correlation between the assets, quantities that only become available (if then!) with time.

In order to measure the correlation between assets, we use the variance-covariance matrix **C** based on a dataset from EURO-STOXX market sector indices. The covariance matrix is updated daily and the individual covariances thus calculated at time T and over a time horizon M are given by (see for example [Litterman & Winkelmann(1998)]):

$$\sigma_{ij}^{T}(M) = \frac{\Sigma_{s=0}^{T} w_{T-s} r_{i,T-s} r_{j,T-s}}{\Sigma_{s=0}^{T} w_{T-s}} \tag{1}$$

where $r_{i,T}$ is the daily return on the ith asset at date T and w_T is the weight applied at date T.

For our purposes, we use asset (i.e. index) price instead of return, $r_{i,T}$ and take unit weights for previous days' data (i.e. $w_{T-s} = 1$ for all s,T) in the calculation of covariances.

As regards volatility measurement, it has been known for some time (see e.g. [Bouchaud & Potters(2000)]) that the eigenstates of the correlation matrix of assets are useful in the estimation of risk in a portfolio made up of those assets. In this paper, we suggest using the *covariance* matrix **C** because, as we will show, this seems to contain more long term information on co-movements of stocks as the covariances retain volatility information which is lost in the normalisation process. Furthermore, the need to use correlation does not apply for indices. For volatility, we suggest the use of day-to-day (or period-to-period) change in the largest eigenvalue of the covariance matrix as a measure of the change in the riskiest position as perceived by the majority of market participants. This, we believe to be novel; up to now, while it has been recognised that the largest eigenvalue (λ^T_{max}) and corresponding eigenvector (z^T_{max}) represented "the market" (with the eigenvector representing in some way the riskiest portfolio), the change in λ^T_{max} over time has not (to our knowledge) been used before as a barometer of the perception of risk in the market.

Specifically, we define the *change in cohesion*, $\Delta \mathsf{C}_{T+1}$ as being

$$\Delta \mathsf{C}_{T+1} = \frac{\lambda^{T+1}_{max}}{\lambda^T_{max}} \qquad (2)$$

3 RESULTS AND DISCUSSION

3.1 Epochs: Definition and Possible Causes

We have taken as our data, daily market sector index prices from EURO-STOXX covering 18 different market sectors[1] and, starting with a time horizon of 200 days, have calculated the covariance matrices for successive days as per Equation 1. By calculating the ratio of the change in largest eigenvalue λ_{max} over successive time periods, and plotting this against the price of the particular market sector index, we obtain a view of how the market is tending to organise itself. Referring to Figure 1, which shows the daily change in λ_{max} for the telecom and technology indices, we see evidence for the epochs mentioned above. Central to the idea of the epochs is the notion that "there are many ways for a market to go up but just one to go down". Various authors ([Zumbach et al.(1999)] and others) have identified key events which can have an (occasionally disproportionate) impact on financial markets. Similarly we have identified events which have acted as market breakpoints and can be clearly identified as marking the end of epochs. Some of these are given in Table 1.

Table 1. Key Events as Breakers in Market Cohesion

Date	Event
07/97 - 11/97	Asian Market Crisis
14/09/98	Bad News from South America
23/07/99	Plunge after Market Highs & Greenspan Address
12/12/00	US Supreme Court Judgement Ratifies President Bush
28/03/01	Cut in Federal Reserve Rate
18/09/01	Post September 11th Reaction

It should be emphasised, however, that not all these events appear to be seen by the market as bad *per se*. Often, it seems to be just a pause for breath or a lack of any particular cohesion. Successive epochs seem to be characterised by progressively higher slopes in price change for an increased degree of cohesion, each epoch being brought to an end by a particular event. Finally, it would

[1] c.f. www.stoxx.com

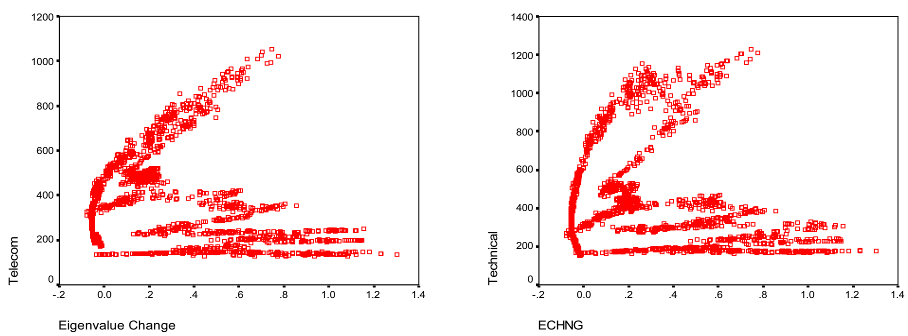
appear that, as extreme prices are reached, further increases in cohesion become impossible and an avalanche will occur with cohesion and price both falling together.

3.2 Epochs: Models and Breakdown

From Figure 1, it will be apparant that cohesion graphs such as those shown, are characterised by periods of simple linear correlation between price and changes in cohesion. Fitting a simple linear model of the form:

$$Y_T \;=\; \alpha_T + \beta_T X_T + \epsilon_t \qquad (3)$$

(where α_T, β_T are functions of time, Y_T are the daily prices, X_T, the daily change in cohesion and ϵ_t are the errors) we get a good fit ($R^2 \approx 0.45 - 0.66$) for most market sectors. However, ϵ_t, which should be independently distributed, actually follows a random walk. There is clearly, therefore, time-dependence of order one in ϵ_t with partial auto-correlation function (PACF) at lag one very close to one and a Durbin-Watson[2] (DW) statistic d very much less than 2.



(a) Telecoms Sector (b) Technology Sector

Fig. 1. Daily Change in Cohesion ($\lambda_{max}^{T+1}/\lambda_{max}^{T}$) vs Price for Different Market Sectors

Hence, this implies we should take first differences of Equation 3:

$$\Delta Y_T \;=\; \beta + \gamma \Delta X_T + \delta \Delta (T X_T) \qquad (4)$$

(where β is small and positive and γ, δ are large and (negative, positive) respectively). Re-examining the modified model we find that the DW statistic is satisfied: $d \approx 2$ so this is now consistent with the null hypothesis of no positive autocorrelation and with no remaining systematic pattern in the residuals (white noise). However, at the *daily* sampling rate, the model fit becomes very poor after taking first differences. Examining different sampling rates (see Table 2), the degradation in R^2 can be clearly seen for the technology market sector index.

[2] testing for positive autocorrelation in the residuals

Table 2. Model Fit Statistics for Technology at Different Sampling Rates

Sampling Rate	R^2 Statistic	DW Statistic
Monthly	0.46	1.97
Every 10 Days	0.40	2.05
Every 5 Days	0.40	2.20
Every 2 Days	0.20	2.05

This data suggests that the variability (or volatility) increases markedly as the resolution increases and conversely that there is a settling period after which short term effects do not have an impact on subsequent data.

3.3 Possible Reasons for Model Breakdown

As we have seen above, the simple linear regression models break down for fine granularity of sampling. There are several possible reasons for this

- *Day of the week.* It seems possible that market behaviour on certain days of the week, is subject to increased/reduced activity, reflecting start-up, closure and similarly. We rejected this explanation, however, as eigenvalue changes and hence changes in cohesion were highly variable for all same-day data without exception.
- *Statistical leverage or outliers.* This presumes that extreme values occurring at certain time points or reflecting abnormally large localised price fluctuations are peculiar to a high sampling rate. This does not seem to be the case, since even very coarse-grained sampling, (i.e. monthly), retains the epoch skeleton. Furthermore, while epoch turning points are clearly significant, the fit is satisfactory for lower sampling rates, but not for high ones. Similarly, outlying points appear to have little influence on the R^2 or Durbin-Watson statistics for all rates and, in any case, are relatively few in number. It does seem likely, however, that there exists a settling period in the market, a few days in length, during which high variability diminishes the fit.
- *Other systematic features.* Even at the daily rate, (fine-grained sampling), first-order filtering satisfactorily results in white noise. Unfortunately, the relatively poor level of fit achieved for these higher sampling rates indicate that the model is significantly less reliable in terms of predictability value.

What does seem clear is that the increased variability over a few days, reflects high volatility associated with changes in cohesive or co-operative behaviour, with frequent changes of the sign of the largest eigenvalue; (the linear trend is in any case strictly non-monotonic). It is, therefore, instructive to consider the nature of the volatility measure in more detail.

Clearly, implied/price volatilities will change *no matter what* so that the historical basis should be relatively bias free. Equally, λ_{max}^{T} is an effective measure of volatility, so that the change $\Delta \mathsf{C}_{T+1}$ in the coherence is equivalent to looking at proportional change in volatility. Typically, these high-frequency fluctuations are self-correcting, representing a short-term and not particularly far-reaching lack of cohesive market behaviour, i.e. $\Delta\mathsf{C}_{T+1} \approx 1$. In other words, if coherence represents the change in commonly-perceived risk, deviation from the common perception is sufficient to ensure fluctuating variability over the short-term and corresponding reduction in fit, as observed for our data.

While this position is usually resolved over a slightly longer period, it is the lack of a sustained common perception of risk, (i.e. lack of coherence), which leads to a drop in price. The quantity $\Delta\mathsf{C}_{T+1}$ is clearly large and negative at the turning point of an epoch, corresponding to large change in the maximum eigenvalue, as well as a change of sign, since no change of sign for λ^{T}_{max} between consecutive time points implies that C is incremented. Unfortunately, since high variability is naturally associated with short-term effects, as mentioned previously, it is non-trivial to determine whether a sustained change can be determined in advance. A number of points are worth noting, however. Firstly, use of the covariance measure exposes the detail of these changes. The epochs, while present to some extent in the correlation matrix information, are far less distinct and thus less useful for our purposes, while dimensionality considerations do not arise for indices. Secondly, we note that outliers are few, so that tolerances can more readily be established on the size of high-frequency fluctuations. (It seems clear that the price change distributions involved do not scale simply, since volatility is dependent on the time interval as usual). It is suggested that coherence provides an intermediate measure which can be directly linked to critical market uncertainty.

4 Conclusions

We demonstrate the existence of epochs in EURO-STOXX market sector data, where the change in the largest eigenvalue of the covariance matrix of daily prices, varies linearly with time. The evidence supports an implicit relationship between instantaneous volatility and the change in the maximum eigenvalue, but volatility patterns are, typically, dependent on the time intervals observed. At the end of an epoch, the relationship changes, with smaller eigenvalue changes leading to the same price change. Epochs are present in all market sectors, but are most strongly defined in the less-defensive sectors, such as technology, telecoms. The epoch end is followed by a return to low coherence in market trading, due to disparate perception of risk. Co-operative behaviour (or strong coherence) in market trading suggests that up-turn patterns are more varied and hence less easy to predict, whereas down-turn patterns are well-defined, due to a common perception of risk change. As a final note, it should be mentioned that we attempted to replicate the results for exchange rate data but as data were limited, partial support only was obtained for results reported here.

5 Acknowledgements

The authors would like to acknowledge Dublin City University and the Institute of Numerical Computation and Analysis (INCA) for financial support for travel and Dr. Michael Mura for his invaluable contributions and insights.

References

[Elton et al.(2002)] Elton, E.J., Gruber, M.J., Brown, S., Goetzmann, W.: Modern Portfolio Theory and Investment Analysis John Wiley and Sons (2002)

[Litterman & Winkelmann(1998)] Litterman, R., Winkelmann, K.: Estimating Covariance Matrices, in *Goldman-Sachs Risk Management Series*, Goldman, Sachs & Co. (1998)

[Bouchaud & Potters(2000)] Bouchaud, J.-P., Potters, M.: Theory of Financial Risks - From Statistical Physics to Risk Management, Cambridge University Press (2000)

[Zumbach et al.(1999)] Zumbach, G.O., Dacorogna, M.M., Olsen, J.L., and Olsen, R.B.: Introducing a Scale of Market Shocks, O & A Research Group Preprint GOZ.1998-10-01, (1999)