

CA313 Algorithms and Complexity

Class Test 2

Attempt **all** questions. All questions carry equal marks. You have **40 mins.** to attempt these questions.

Q 1.

Assume the following Type 0 grammar:

$$\langle \begin{array}{l} V_t = \{a, \textit{computer}, \textit{Peter}, \textit{sold}, \textit{is}, \textit{was}, \textit{by}, \textit{faulty}\} \\ V_n = \{S, NP, VP, V, D, N\} \\ P = \{S \rightarrow NP, VP \\ \quad NP \rightarrow D, \textit{computer} \\ \quad NP \rightarrow \textit{Peter} \\ \quad VP \rightarrow V \\ \quad VP \rightarrow V, NP \\ \quad D \rightarrow a \\ \quad V \rightarrow \textit{sold} \\ \quad \textit{Peter sold a computer} \rightarrow \textit{the computer was sold by Peter} \\ \quad \textit{a computer sold Peter} \rightarrow \textit{Peter was faulty}\} \\ S = S \rangle \end{array}$$

(i) Write down the strings in the language permitted by this grammar.

(ii) Show which of the rules in the grammar could be rules in a CSG, a CFG, or an FSG. Explain why.

(iii) Which rules in the set P make this grammar a Type 0 grammar? Explain why.

Q 2.

(i) Define the constraints on α and β in a context-sensitive grammar.

(ii) Assuming the following ruleset:

- $W \rightarrow xyz$
- $W \rightarrow xWYz$
- $zY \rightarrow Yz$
- $yY \rightarrow yy$

(a) what are the first three strings in this language?

(b) Show the derivation of the 2nd shortest string using trees;

(ii) Show the derivation of the 3rd shortest string using string manipulation.

(c) Does it make a difference in which order you apply the rules? Explain your answer.

Q 3.

Describe in your own words:

(a) What a Turing machine consists of?

(b) What it is useful for?

(c) Give a formal definition of a Turing machine in terms of the 5-tuple $M = (Q, \Sigma, \Gamma, q_0, \delta)$.

Q 4.

(i) Assuming unary input, describe the transition function of the Turing machine **UN-1**.

(ii) Comment on the approach you have taken compared to other possible solutions.

(iii) For the initial configuration $(q_0, \underline{1}11\#)$, show how the Turing machine you provided in (i) copes with that input tape.

(iv) If you were able to conceive of a different Turing machine in (ii), show how that copes with the same input tape.