Another Proof by Induction

For example, let's prove:

$$P(n) \geq 8 \Longrightarrow n\ can\ be\ written\ as \sum x\ 3's + y\ 5's$$

Base case is $n = 8 : 3 + 5$

NB, If $n \leq 7$, proof fails.

Note that:

$n = 9 : 3 + 3 + 3$

$n = 10 : 5 + 5$

Let's assume that $P(8), P(9), P(10) \ldots P(n)$ are all true.

For the inductive step, we must prove $P(n+1)$.

Strategy: let's subtract 3 from $n+1$, observe that this number can be written as a sum of 3's and 5's, and add one more 3 to find a way to write $n+1$.

We know that $n-2 \geq 8$, so we may assume $P(n-2)$.

That is, $n-2 = 3a + 5b$ for some integers a and b.

Therefore, $n+1 = 3 + 3a + 5b$, so $n+1$ can be written as the sum of $a+1$ 3's and b 5's.

That proves $P(n+1)$, and concludes the inductive step. $\square$