CA313 Algorithms and Complexity
Autumn 2008

Attempt **three** questions. All questions carry equal marks.

Q 1.
Assume the following Type 0 grammar:

$\langle \quad V_t = \quad \{a, house, Martin, bought, is, was, by, happy\}$
$\quad V_n = \quad \{S, NP, VP, V, D, N\}$
$\quad P = \quad \{S \rightarrow NP, VP$
$\quad\quad\quad\quad NP \rightarrow D, house$
$\quad\quad\quad\quad NP \rightarrow Martin$
$\quad\quad\quad\quad VP \rightarrow V$
$\quad\quad\quad\quad VP \rightarrow V, NP$
$\quad\quad\quad\quad D \rightarrow a$
$\quad\quad\quad\quad V \rightarrow bought$
$\quad\quad\quad\quad Martin\ bought\ a\ house \rightarrow a\ house\ was\ bought\ by\ Martin$
$\quad\quad\quad\quad a\ house\ bought\ Martin \rightarrow Martin\ was\ happy\}$
$\quad S = S\rangle$

(i) Write down the strings in the language permitted by this grammar. Show their derivations using trees.

(ii) Show which of the rules in the grammar could be rules in a CSG, a CFG, or an FSG. Explain why.

(iii) Which rules in the set P make this grammar a Type 0 grammar? Explain why.

Q 2.
(i) With respect to *languages*, give definitions for the following terms:

- an *alphabet*;
- a *string*;
- the *length* of a string;
- the *empty string*.

(ii) Let us refer to an alphabet as Σ. Explain, therefore, the following:

- Σ^*
- Σ^+

(iii) Give a definition of a *language*, with respect to Σ^*.

(iv) We define a *grammar* in terms of the quadruple $\langle V_t, V_n, P, S \rangle$. Explain each of these terms.

(v) For each of the four grammar types in the Chomsky Hierarchy, give the:

- class of grammar name;
- language name;
- automaton

associated with each type.

Q 3.
(i) Define *time complexity*.

(ii) Exponential solutions are usually considered inefficient, while polynomial solutions are considered efficient. Give examples which show that this is not completely true.

(iii) Name the following complexities (e.g. $O(n^3)$ is 'cubic'), and for each of them, say if they are usually considered efficient (e.g. cubic complexities are considered efficient only for small values of n).

- $O(5)$
- $O(n)$
- $O(exp(n))$
- $O(n^k)$ (k is a natural number)
- $O(ln(n))$
- $O(n^2)$
- $O(3^n)$

(iv) Write the following functions using the O notation (e.g. $5n = O(n)$).

- $100n^2$
- $3ln(n) + 100$
- $exp(n) + 2n^3$
- $4n^3 + 250n^2$
- $2n^2\ x\ ln(n)$

Q 4.
(i) Define the following classes of complexities:

- P,
- NP,
- NP-hard,
- NP-complete.

(ii) Explain the Traveling Salesman Problem (TSP). Give three different applications of the TSP.

(iii) Explain why we need to apply heuristics when confronted with a problem in the complexity class NP.

(iv) Describe **one** algorithm that leads to an approximate solution for NP in reasonable time.

(v) Give **one** example of a heuristic that you might employ in a game-playing situation.

Q 5.
(i) Define *space complexity*.

(ii) Explain *time complexity* and *space complexity* with respect to two core components in the computers we use every day.

(iii) What is the difference between the two classes $DSPACE$ and $NSPACE$?

(iv) Describe clearly the relationship between the time complexity classes P, NP, and *co-NP*, and the space complexity class $PSPACE$.