

CA313 Algorithms and Complexity

Autumn 2007

Attempt **three** questions. All questions carry equal marks.

Q 1.

(i) With respect to *languages*, give definitions for the following terms:

- an *alphabet*;
- a *string*;
- the *length* of a string;
- the *empty string*.

(ii) Let us refer to an alphabet as Σ . Explain, therefore, the following:

- Σ^*
- Σ^+

(iii) Give a definition of a *language*, with respect to Σ^* .

(iv) We define a *grammar* in terms of the quadruple $\langle V_t, V_n, P, S \rangle$. Explain each of these terms.

(v) For each of the four grammar types in the Chomsky Hierarchy, give the:

- class of grammar name;
- language name;
- automaton

associated with each type.

Q 2.

Assume the following Type 0 grammar:

$$\langle \begin{array}{l} V_t = \{the, newspaper, Mary, bought, was, by, damaged\} \\ V_n = \{S, NP, VP, V, D, N\} \\ P = \{S \rightarrow NP, VP \\ NP \rightarrow D, newspaper \\ NP \rightarrow Mary \\ VP \rightarrow V \\ VP \rightarrow V, NP \\ D \rightarrow the \\ V \rightarrow bought \\ Mary\ bought\ the\ newspaper \rightarrow the\ newspaper\ was\ bought\ by\ Mary \\ the\ newspaper\ bought\ Mary \rightarrow Mary\ was\ damaged\} \\ S = S \rangle \end{array}$$

(i) Write down the strings in the language permitted by this grammar. Show their derivations using trees.

(ii) Show which of the rules in the grammar could be rules in a Type 1, Type 2, or a Type 3 grammar. Explain why.

(iii) Which rules in the set P make this grammar a Type 0 grammar? Explain why.

Q 3.

(i) Define the constraints on α and β in a Type 0 grammar rewrite rule, where a rule is of the form $\alpha \longrightarrow \beta$.

(ii) What are the implications for the learning of such a language from input alone?

(iii) Give a formal definition of a Turing Machine in terms of the quintuple $M = (Q, \Sigma, \Gamma, q_0, \delta)$.

(iv) Construct a Turing Machine which erases the input string.

(v) For the initial configuration $(q_0, \underline{1}11\#)$, show how the Turing machine you provided in (iv) copes with that input tape.

Q 4.

(i) Give a formal definition of time complexity.

(ii) Give a formal definition of space complexity.

(iii) What is an NP problem?

(iv) What is an NP-complete problem?

(v) Give an example of an NP problem and of an NP-complete problem.

Q 5.

(i) What are heuristics?

(ii) In which context can they be used?

(iii) Give two examples of heuristic methods.

(iv) Describe the main principles of the heuristic methods you mentioned in (iii).